Geometrical Optical Analysis of Defocused Retinal Images to Compute the Size of Retinal Blur Circles Relative to Object Size

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Abstract

Optical analysis of the blurred retinal image for single points of light reveals simple relationships between the amount of blur, the amount of defocus and/or astigmatism, and the pupil diameter. The same rules apply to extended objects that are treated as a collection of point sources. Although the angular size of blurred retinal images of extended objects is distorted slightly by ocular astigmatism, it is relatively unaffected by spherical ametropia.

Keywords

Optical blur, defocus, astigmatism, retinal image

From an optical perspective, everyday objects are a myriad of points of light-emitting rays that, after penetrating the pupil of the eye, form an image on the retina. According to the laws of geometrical optics, when the optical system of the eye is well focused, each point of light in the object forms a point of light in the retinal image. In reality, the image is not a simple point, because the physical factors of diffraction and interference distribute the light across the retina. The result is a complex pattern of light that requires the sophisticated tools of Fourier analysis to describe it fully mathematically. Although those physical factors are taken into account in the physical optics theory of image formation, they are ignored in the simpler geometrical optics theory of image formation, which is the basis for the following discussion.
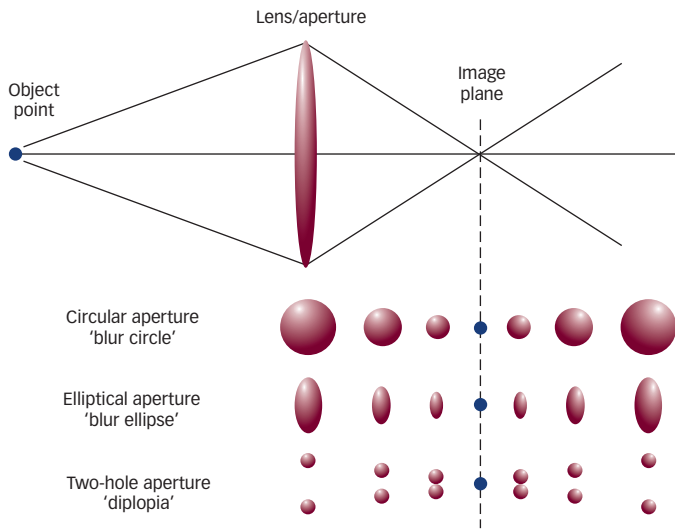
Point Objects

When the optical system of an eye is mis-focused on an object, the image of any single point of light is uniformly spread out across a small area of retinal surface. As illustrated in *Figure 1*, the shape of the pupil of the eye determines the shape of the blurred retinal image. Given that the shape of the pupil in the normal human eye is circular, the image is a circular region called a 'blur circle' or 'blur disk'. By comparison, the eye of a cat has a vertically elongated pupil, so the retinal image would be a 'blur ellipse'. The human pupil also takes on an elliptical appearance when viewed from the side, so the blurred image in peripheral retina is also a 'blur ellipse'. Some animals have a pupil that forms two small pinholes, which would produce a pair of small blur disks for every object point, a natural example of monocular diplopia.

Regardless of the shape of the pupil, the blurred image of a point source grows larger in proportion to the amount of defocus. This effect is easily proven by the geometrical optical analysis presented in *Figure 2*. For this purpose, it is sufficient to use a simple optical model of the eye comprising a single refracting surface, the so-called 'reduced eye' model. The curved refracting surface separates the ocular medium (with refractive index n) from air (with refractive index 1.0). The example shown in *Figure 2* is for an uncorrected myopic eye viewing a distant point of light, such as a star in the night sky. The pupil has diameter d and the rays of light come to focus at a distance f from the cornea. For a myopic eye, the focus point lies some distance z in front of the retina. The focusing error E can be quantified as the difference between the dioptric power of the eye (n/f) and the dioptric length of the eye $(n/(f+z))$. Thus, $E = n/f - n/(f+z)$ has the units of diopters when f and z are specified in meters. For small amounts of defocus, z is small compared with f, which enables an algebraic simplification of this formula to yield the approximate relationship $E = nz/f^2$. From the geometry of similar triangles made by the marginal rays, we know that $b/d = z/f$. Multiplying both sides of this equation by n/f gives $nb/df = nz/f^2$, which also equals E, according to the above equation. The final step in this analysis is to interpret the blur circle diameter b as a visual angle subtended at the nodal point of the eye (see *Figure 2*). This angle, measured in radians, is equal to the blur disk diameter divided by the distance from the retina to the nodal point. Again assuming that z is small compared with f, the blur circle diameter β in radians is equal to $b/(f/n)$. Substituting this result into the previous equation yields the final approximate relationship, $\beta = dE$. Thus, one can conclude that the size of the blur disk (in radians) produced by a single point of light is approximately equal to the pupil diameter (in meters) multiplied by the amount of defocus (in diopters). The formula is valid for positive and negative values of E (i.e. for myopic and hyperopic blur, respectively) if one ignores the sign of the resulting angle β.

Figure 1: Effect of Aperture Shape on the Defocused Image of a Point Source


The aperture shapes the bundle of light rays, thereby determining the shape of the patch of light found in front of, or behind, the image plane.

Extended Objects

Everyday objects extend beyond the size of a single point of light and, therefore, the retinal image is extended even when well focused. When mis-focused, the image is slightly larger because each point of the image is blurred in proportion to the amount of blur and the pupil diameter, as described above. To understand the relative importance of object size and blur circle size in determining the final image size, it is helpful to consider extended objects that comprise only a few points of light. For example, the constellations of stars in the night sky form outlines of objects for which the overall dimensions are easily determined, even when blurred by defocus. One such constellation, the great bear Ursa Major (including the big dipper), is shown in *Figure 3*. Although the physical distance between individual stars in the constellation are astronomically large, the visually important dimension is their angular separation, which can be specified approximately by the 'Rule of 60s', explained in *Figure 3*. One radian is the angle subtended at the center of a circle by an arc with length equal to the radius of the circle. So, if the radius is 60 units long, an arc of length 60 will subtend 1 radian, which is approximately equal to 60 degrees (the exact value is $180/\pi$, which rounds up to 60). Thus, each unit of distance along the arc subtends 1°. For small angles, a circular arc can be approximated by a straight line, which means that one can estimate the angular separation between two stars by holding a centimeter ruler at arm's length (approximately 60 cm) and aligning the stars to the markings on the ruler. Using the conversion factor 1 cm = 1°, the separation between the two stars Dubhe (Arabic for 'bear') and Merak (Arabic for 'flank of the bear') in Ursa Major is shown to be 5°. These two stars (also known as the 'pointers', because they point to Polaris, the North Star) calibrate photographs of Ursa Major so that other angular distances can be determined.

Figure 4 simulates the effect of defocus on the retinal image of an extended object. A clearly focused photograph of Ursa Major and Ursa Minor is shown in *Figure 4A* and a blurred photograph of the same

Figure 2: Geometry of Image Blur
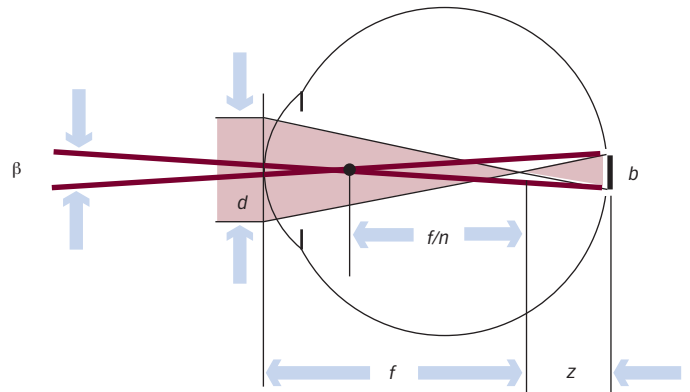
A defocused point source produces a blur patch on the retina with linear diameter b, which subtends angle β at the nodal point of the eye (dot). The shaded area shows a beam of light entering the eye; red lines show the angular subtense of the blur disk.

Figure 3: Measuring Object Size as a Visual Angle
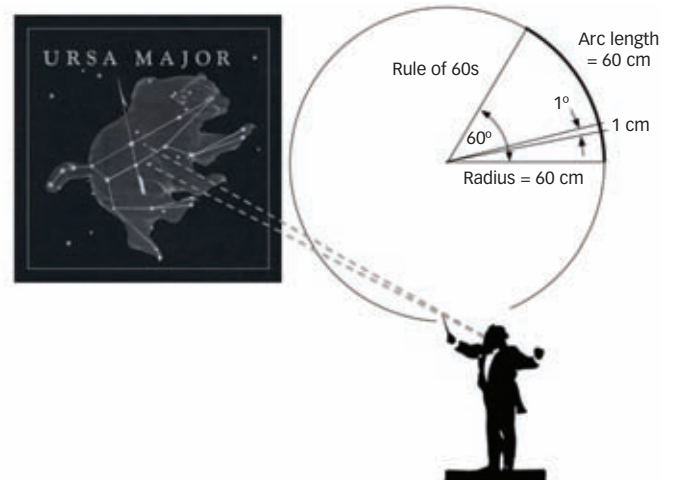
A ruler held at arm's length can be used to measure the size of an object using the ' Rule of 60s'—if the ruler is 60 cm from the eye, then an arc 60 cm long subtends approximately 60 degrees of visual angle; thus, 1 cm = 1°.

object is shown in *Figure 4B*. Using the scale bar as a ruler, one can see that defocus has blurred the individual stars in the constellation, but has no effect on the center-to-center separation of the stars. This is because defocus is a symmetrical form of blur that spreads the light from any single point source into a blur disk centered on the clearly focused image. The scale bar indicates that the blur disk diameter is approximately 1.5°, or 0.026 radians. Thus, the photograph in *Figure 4B* simulates the retinal image in an eye for which the product of pupil diameter and defocus is 0.026. For example, if the pupil diameter is 6.5 mm, then the photograph simulates four diopters of defocus. Given that the overall size of the object in this example is much greater than the size of the blur disks, blurring has a negligible effect on the size of the blurred image. This result is typical of extended objects. Blurring affects image size only when the angular dimensions of an extended object are of the same order of magnitude as the size of the blur disk. For example, the same amount of blur would greatly enlarge the image of the moon, because the angular size of the latter (0.5°) is comparable to the size of the blur disk (1.5°) for individual points of light.

Figure 4: Photographic Images of the Big Dipper (Ursa Major) and the Little Dipper (Ursa Minor) when well Focused (A), and when the Image is Defocused (B)
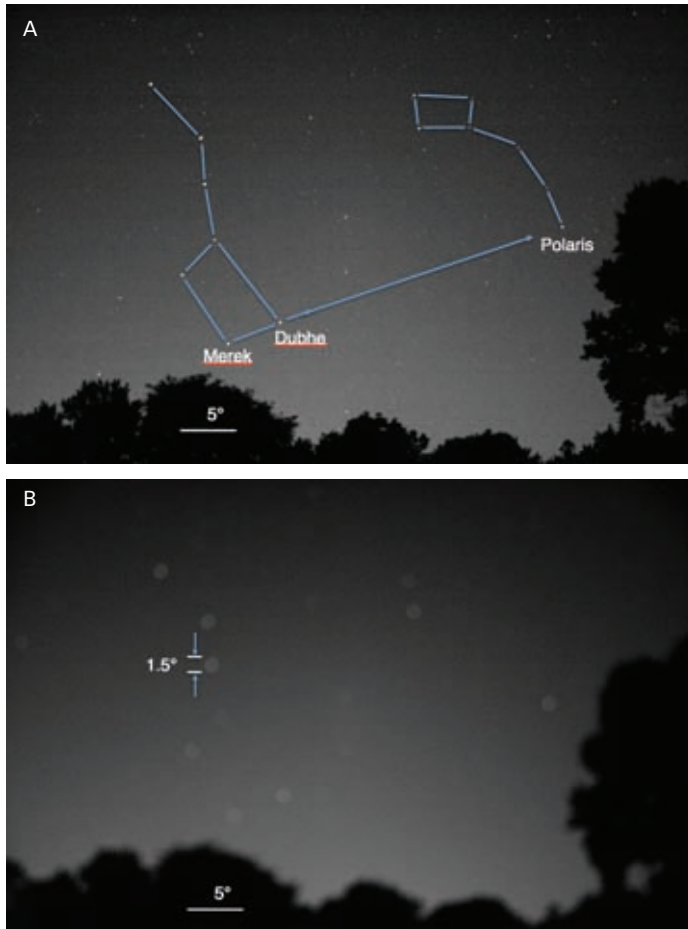

Defocus expands the images of individual stars but does not affect the spacing between stars. The angular calibration bar is based on the known separation of the 'pointer' stars Dubhe and Merek.

Astigmatic Blur

Ocular astigmatism has two effects on the retinal image—it distorts and blurs the retinal image. Distortion is to the result of small differences in the axial location of the entrance pupil of the eye. The entrance pupil of any optical system is the apparent limiting aperture seen from the viewpoint of the object. The entrance pupil is a useful construct because any ray traveling in a direction that will allow it to pass through the entrance pupil before refraction will also pass through the physical pupil after refraction. For the reduced eye model, the entrance pupil is the

Figure 5: Geometry of Image Distortion in the Astigmatic Eye
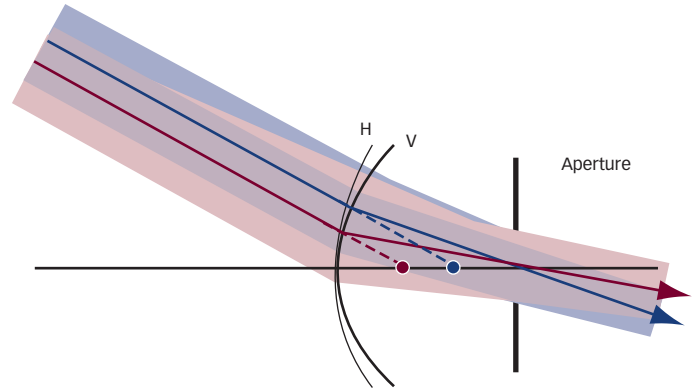
In this example, the vertical meridian has greater optical power, which causes the entrance pupil to appear closer to the physical aperture, compared with the horizontal meridian. As a result, a vertical sheet of rays (blue) from a distant object emerge from the aperture, traveling in a slightly different direction relative to a horizontal sheet (red). Chief rays (indicated by arrowheads) pass through the aperture center and locate the centers of the blurred retinal images. H = horizontal meridian; V = vertical meridian.

image of the iris aperture formed by the refracting surface. An astigmatic surface is characterized by a focal power that varies sinusoidally with the meridian. The maximum and minimum powers occur in two orthogonal, principal meridia. *Figure 5* illustrates an example of a model eye that has 'with the rule' astigmatism. The refracting surface of the model has greater optical power in the vertical meridian than in the horizontal meridian and, consequently, the entrance pupil for a vertical sheet of light lies slightly further from the refracting surface compared with the entrance pupil for a horizontal sheet of light. Consequently, these two sheets of light emerge from the aperture traveling in slightly different directions and they strike the retinal at slightly different lateral distances from the optical axis. This distortion is very small in typical human eyes, being less than 1 % for 3D of astigmatism.

The shape of the blur disk on the retina of the astigmatic eye depends not only on the amount of astigmatism, but also on the amount of spherical refractive error present. The simplest situation to analyze is when the mean spherical equivalent of the refractive error is zero. In this case, the refractive error E is equal in magnitude, but opposite in sign, for the two principal meridia. According to the optical analysis performed above, the size of the blur disks will be the same in both principal meridia and, thus, circular in shape. The formula $\beta = dE$ remains valid, with the focusing error E equal to half the cylindrical component of the refractive error. ■